Optimal Incentives in Three-Level Agent Systems

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Abstract

Should superiors give incentives to subordinates to motivate good work, and how large should the incentive be? In this paper, we explore optimal incentive payments and the conditions under which a superior would offer a share of her income to subordinates. We model and analyze the interaction between three hierarchically interacting agents using multiscale decision theory. We determine the participation constraint, which is the condition under which a superior agent is willing to provide an incentive. A spreadsheet based decision tool is presented that supports superiors in deciding if and how much incentives to offer to lower level agents.

Keywords
Multiscale decision theory, decision analysis, game theory, organizational design, incentives

1. Introduction

Effective incentives motivate employees to work in the interest of their superiors, i.e., behave cooperatively[1]. The organizational structure of an enterprise describes the superior-subordinate relationship. Most companies are structured hierarchically and a superior's success depends on the actions of lower level decision makers. Consequently, higher level agents will consider offering a reward or profit sharing plan to their subordinates to ensure that subordinates are making decisions in the superior’s interest. Contributions to incentive research come from various fields: management systems, economics, and control theory (see [2-4] for several examples). The question of when and where incentives should be applied is an interesting problem for academics and practitioners[5, 6]. In this paper, we investigate when it is advantageous for a superior to offer incentives to lower level agents. We investigate a three-level system in which agents seek to maximize their personal rewards. The organizational structure of our system is based on the multiscale decision making model Wernz and Deshmukh[7, 8]. Multiscale decision theory uses reward sharing to align the interests of hierarchical agents. Multiscale decision theory presents an approach for analyzing multi-organizational-scale decision making problems in which a multi-agent hierarchy is represented as a stochastic agent model. The authors solved a coordination problem between two agents in a manufacturing enterprise challenged and extend the model to multiple levels[7]. Wernz and Henry apply multiscale decision theory to solving a three-level service enterprise problem[9]. Their problem concerns a maintenance service division made up of an account manager, maintenance supervisor, and maintenance worker. We investigate the participation constraint in a three-level system, which determines whether the top-most agent wants to provide incentives to induce cooperative behavior by subordinate. They show that for a given numerical example all agents in the system can benefit from an incentive plan. The participation constraint defined in this paper provides a functional solution for when incentives provide benefit to the overall system. To help real-world decision makers faced with this type of problem, we have developed a Participation Calculator that informs agents when to offer or accept an incentive, how high the incentives should be provided and how much participants benefit from providing and accepting incentives.

The remainder of the paper is organized as follows: Section 2 introduces the multiscale decision model of Wernz and Deshmukh and its application to service enterprise systems. We follow this with our analysis of the
participation constraint and introduce the Participation Calculator in Section 3. We then present insights, highlight areas of future work, and concluding remarks in Section 4.

2. Model Introduction

In this paper we address the questions surrounding the application of incentives in a three-agent system consisting of agents A1, A2, and A3 organized in a hierarchy. Each agent is responsible for making a decision between two possible actions from their respective actions sets simultaneously

\[ A^1 = \{a_1^1, a_2^1\}, \quad A^2 = \{a_1^2, a_2^2\}, \quad A^3 = \{a_1^3, a_2^3\}. \]  

(1)

Following the action, each agent will transition to just one of two possible states in their respective state space

\[ S^1 = \{s_1^1, s_2^1\}, \quad S^2 = \{s_1^2, s_2^2\}, \quad S^3 = \{s_1^3, s_2^3\}. \]  

(2)

The probability of transitioning from a given action to a particular state can be represented using an initial transition probability vector

\[ \alpha^x = \begin{bmatrix} \alpha_{1x}^x \\ \alpha_{2x}^x \end{bmatrix}, \quad x \in \{1, 2, 3\}. \]  

(3)

where \( \alpha_{1x}^x \) represents agent \( Ax \)'s conditional probability of ending up in state with index \( j \) given it takes action with index \( j \) and \( \alpha_{1x}^x \) is a non-negative probability less than or equal to 1. Using law of total probability \( 1 - \alpha_{1x}^x \) represents the probability of taking action \( j \) and ending up in any state with index other than \( j \). In the two outcome case \( 1 - \alpha_{1x}^x \) is the probability of action \( 1 \) resulting in state \( 2 \). Each agent is aware of its personal transition probability vector prior to making a decision.

Upon arriving in a given state each agent receives a state dependent reward \( r^x(s^x) = \rho^x_j \). We can also write this in vector form where \( \tilde{\rho}^x \) is the vector of possible state dependent rewards for agent \( Ax \).

\[ \tilde{\rho}^x = \begin{bmatrix} \rho_{1x}^x \\ \rho_{2x}^x \end{bmatrix}, \quad x \in \{1, 2, 3\}. \]  

(4)

The transition probability vectors and reward vectors (3) and (4) are known privately to each agent before a decision is made and represents the structure prior to any agent interaction or incentives. Hierarchical systems have outcome dependencies. Multiscale decision theory models this outcome dependency using a transition probability function. agent A3’s effect on agent A2 is represented by the transition probability function \( f_2 \) which is of the form,

\[ f_2(s_2^3, s_2^2) = \begin{bmatrix} c_2 \\ -c_2 \end{bmatrix}. \]  

(5)

Where the rows are linked to agent A3’s outcomes and columns to possible the outcomes of agent A2 The value of the transition probability function \( c_i \) is a non-negative constant, which is either added to or subtracted from the initial transition probability vector of agent A2, \( \tilde{\rho}^2 \). The influence of agent A2 on agent A1 is of the same form

\[ f_1(s_2^2, s_1^1) = \begin{bmatrix} c_1 \\ -c_1 \end{bmatrix}. \]  

(6)

The initial transition probability vector of agent A1 is affected by agent A2’s state which is influenced by agent A3’s state and thus agent A1 is indirectly affected by agent A3. It should be noted that the influence of an agent on another agent is a property of the system thus it cannot be altered by an agent.

The final transition probability of agent A1 is

\[ p_{final}^1(s_1^1 | a_1^1, a_2^2, s_2^3) = p^1_1(s_1^1 | a_1^1) + f_1(s_1^1, s_2^2) \cdot \left(1 + f_2(s_2^2, s_2^3)\right). \]  

(7)
In response to this bottom-up outcome dependency, a top-down reward sharing incentive policy is proposed. Multiscale decision theory borrows notation from Dolgov and Durfee to represent the agent interactions graphically in a dependency graph (Figure 1)[10]. The dashed arrow going from agent A1 to A2 represents the reward sharing incentive paid by agent A1 to A2. Likewise agent A2 provides a share of her total reward with A3, illustrated by the dashed arrow from A2 to A3. Agent A3 is indirectly affected by the incentive provided from A1 to A2 in the dependency graph this is represented by the dashed incentive arrow emanating from the incentive arrow between agent A1 and A2. The bottom-up flow of influence is of a similar structure in the opposite direction. Note agents who pay out incentives to lower level agents are doing so out of their own personal reward which is represented in the dependency graph by a dotted arrow.

\[ r_{\text{final}}^{1i}(s_i^{4i}) = r^{4i}(s_i^{4i})(1 - h) \]  

\[ r_{\text{final}}^{12i}(s_i^{41}, s_j^{42}) = \left( 1 - h \right) \cdot r^{41}(s_i^{41}) + h \cdot r^{42}(s_j^{42}) \]  

\[ r_{\text{final}}^{13i}(s_i^{41}, s_j^{42}, s_k^{43}) = r^{43}(s_i^{43}) + b_2 \cdot r_{\text{final}}^{12} = r^{43}(s_i^{43}) + b_2 \cdot r^{42}(s_j^{42}) + b_1 \cdot r^{41}(s_i^{41}) \]  

The term \((1 - h)\) in (8) represents the incentive paid down from agent A1 to A2. Note agent A3’s final reward (10) is a function of agent A1 and A2’s rewards. Agents are assumed to be self-interested and seek to maximize their personal final reward.

Let us assume the payoff structure is in the form show in equation (11) where agent A1 sees the greatest reward from his state \(s_i^{41}\) and thus prefers this outcome state. While all other agents see the greatest reward from state \(s_i^{42}\) and thus initially prefer this state.

\[ \rho_1^{41} > \rho_2^{4i}, \quad \rho_1^{42} < \rho_2^{42}, \quad \rho_1^{43} < \rho_2^{43} \]  

\[ \alpha_m^{4i}, \alpha_2^{42}, \alpha_3^{43} > \frac{1}{2} \]  

For simplicity the constraint in (12) insures that an action and its most probable outcome state have the same index. With (11) and (12) it can be shown that the rational agents’ initial preferences are non-cooperative, where agent A1 prefers his action \(a_i^{41}\) while all other agents prefer their action \(a_2^{4i}\). Multiscale decision theory has shown that rational lower level agents will weakly prefer the cooperative action over their initially preferred non-cooperative
action given the appropriate level of incentive. For example agent A2 will choose the cooperative action if an incentive of level $b_2$ is offered that satisfies the following necessary condition.

$$b_2 \geq \frac{\rho_{31}^{43} - \rho_{41}^{32}}{c_2 \left( \rho_{42}^{21} - \rho_{41}^{31} \right)}$$

(13)

For further detail on this necessary condition, along with a detailed proof see Wernz and Henry[9]. This necessary condition answers the question of whether or not a lower level agent should cooperate and choose the action that is initially not preferred. The decision to offer an incentive or participating in the incentive plan is referred to as the participation constraint or participation decision in multiscale decision theory.

3. Analysis

System wide cooperation increases the probability of top level success. It is known that implementing a reward sharing program in which agent $A(i+1)$ offers agent $A_i$ an incentive plan that meets agent $A_i$’s necessary condition will illicit cooperation. The question remains is an incentive plan beneficial to the upper level agent who is responsible for paying incentives to lower level agents. Since we assume all agents are rational utility maximizing players, upper level agents will choose to offer just enough of an incentive to insure cooperation only if the supremal agent’s expected reward is increased after paying the incentive. Organization wide adoption of a reward sharing program will occur only if

$$r_{final}^{41} \left( s_i^{41}_b | b_{i-1} \right) \geq r_{final}^{41} \left( s_i^{41} \right)$$

(14)

$$r_{final}^{4i} \left( s_i^{4i} \right) \geq r_{final}^{4i} \left( s_i^{4i} | b_{i-1} \right) \quad i \in \{2,3\}$$

(15)

all agents final rewards are at least equal to their rewards before incentives were introduced. Specifically agent A1 for will offer agent A2 a portion of his reward equal to agent A2’s necessary condition

$$b_1^* = \frac{\rho_{42}^{21} - \rho_{41}^{31}}{2c_1 \left( \rho_{42}^{21} - \rho_{41}^{31} \right)}$$

(16)

if his expected reward following paying the incentive and gaining cooperation, is greater than when no incentive is paid and agents act in a non-cooperative manor. From condition (14) we develop the following expected value inequality for agent A1’s participation decision

$$E \left( r_{final}^{41} \left| b_1^*, a_{41}^{41}, a_{42}^{42}, a_{43}^{43} \right. \right) > E \left( r_{final}^{41} \left| b_{i-1} \right. \right)$$

(17)

We chose to solve this inequality for the term $c_1$, the influence agent A2 has on agent A1, resulting in the participation constraint

$$c_1 > f^{41} \left( \tilde{\rho}^{41}, \tilde{\rho}^{42}, \tilde{\alpha}^{41}, \tilde{\alpha}^{42}, \tilde{\alpha}^{43}, c_2 \right)$$

(18)

Where $f^{4i}$ is a function of the system with a closed form result. What the participation constraint (18) tells us is that agent A1 will benefit from offering an incentive to agent A2 if the influence agent A2 has on agent A1 is above $f^{4i}$. This lower bound on $c_1$ represents a system bound in which for values below this participation constraint it is not in the best interest of agent A1 to provide an incentive to agent A2.

Applying the same analysis for agent A2’s decision to offer an incentive to agent A3 yields a similar result

$$c_2 > f^{42} \left( \tilde{\rho}^{41}, \tilde{\rho}^{42}, \tilde{\alpha}^{41}, \tilde{\alpha}^{42}, \tilde{\alpha}^{43}, c_1 \right)$$

(19)

Only if $c_2$, the interaction agent A3 has on agent A2, is greater than $f^{42}$ should agent A2, utility maximizing agent, offer an incentive to agent A3. If both participation constraints are met (18) and (19), multiscale decision theory
dictates that incentives levels $b_1^*$ and $b_2^*$ will be offered by the superior agents A1 and A2 respectively. While these participation functions are large, the development of a spreadsheet-based tool allows decision makers be it agents themselves or an organizational designer to evaluate the participation constraint for a given set of system parameters, and thus answering the incentive participation question for their system.

The Participation Calculator developed in Microsoft Excel is a decision support tool that provides organizational designers and/or agents a simple and effective means by which to analyze the participation constraint for a given system, thus informing the user if a reward sharing program should be implemented. Users enter system parameters into the calculator’s graphical interface which is organized in a hierarchy much like the system which it models. Following the input process, the calculator evaluates $f^{A1}$ and $f^{A2}$ determining if one or both of the participation constraints are met. The $b_1^*$ and $b_2^*$ values are calculated and displayed to the user. The calculator provides the user with an expected benefit for each agent once the prescribed reward sharing plan is implemented. Figure 2 below provides a screenshot of the calculator indicating that both participation constraints were met and the expected benefit from implementing the incentive plan is 56.2, 49.7, 46.4 for agents A1, A2, and A3 respectively.

![Participation Calculator Diagram]

The development of the Participation Calculator provides us with confirmation that decision makers can use multiscale decision theory to answer questions regarding a complex hierarchical system.
4. Conclusion
A closed from solution to the participation constraints along with the developed Participation Calculator allows an organizational designer and/or agent to answer the question, should reward sharing be applied in a three-level multi-organizational-scale system. The participation constraints are programmed into a spreadsheet based Participation Calculator that can be used to aid decision makers with the participation decision. The calculator also provides information about the expected level of benefit from a proposed plan for each agent. The formulation of a participation constraint along with the necessary conditions encourages further mathematical exploration into the system parameter space in which reward sharing incentives lead to system wide benefit. In addition to its use as a decision aid the Participation Calculator can be used as a tool to numerically explore the system parameter space in which incentives are applied. Future research will seek to shed light on the influence each system parameters has on the participation decision. The existence of a closed form solution to the participation constraint along with the development of the Participation Calculator are great first steps in gaining a deeper understanding of the application of incentives in a complex multi-organizational-scale system.